$$\therefore \left.\begin{array}{l} y+x=63 \\ y-x=1 \end{array}\right\} \text{ or } \left.\begin{array}{l} y+x=21 \\ y-x=3 \end{array}\right\} \text{ or } \begin{array}{l} y+x=9 \\ y-x=7; \end{array}$$

whence $y=32$, $x=31$; $y=12$, $x=9$; $y=8$, $x=1$. Since $32=9+23$ and $12=1+11$, 32 belongs to Hendricks, 12 to Klaus, 9 belongs to Katrine, 1 to Gertrude.

$\therefore$ 32 Hendricks, 31 Anna; 12 Klaus, 9 Katrine; 8 Hans, 1 Gertrude.

Hence, Hendricks is Anna's husband, Klaus is Katrine's husband, and Hans is Gertrude's husband.

Also solved by *W. R. LEBOLD*, *G. B. M. ZERR*, and *M. A. GRUBER*.

164. Proposed by JOSEPH V. COLLINS, Ph. D., Professor of Mathematics, State Normal School, Stevens Point, Wis.

Three women, the first with ten eggs, the second with thirty, and the third with fifty, went to market. They each got the same for their eggs, and all returned with the same money. What did they get?

Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

Let a, b, and c=the respective numbers of eggs the three women sold at y cents for every d eggs; and, for the remaining eggs, let x cents=price per egg.

Then $(10-a)x+\frac{a}{d}y=(30-b)x+\frac{b}{d}y=(50-c)x+\frac{c}{d}y$.

Whence, $y=\frac{b-a-20}{b-a}dx=\frac{c-a-40}{c-a}dx=\frac{c-b-20}{c-b}dx$.

Solving for a, b, and c, we find $a+c=2b$. For *positive* values, $a<9$, $b>20+a$ and <31, $c>40+a$ and <51.

Put $d=2$. Take $a=2$; then $b=24$ and 26, $c=46$ and 50, $y=2x/11$ and $\frac{1}{3}x$. For integral values, put $x=11$ and 3, respectively; then $y=2$ and 1. Therefore, 10 eggs brought $\frac{2}{2}\times 2c+8\times 11c=90c$, or $\frac{2}{2}\times 1c+8\times 3c=25c$; 30 eggs brought $\frac{24}{2}\times 2c+6\times 11c=90c$, or $\frac{26}{2}\times 1c+4\times 3c=25c$; and 50 eggs brought $\frac{46}{2}\times 2c+4\times 11c=90c$, or $\frac{50}{2}\times 1c=25c$.

Put $d=3$. Take $a=3$ and 6; then $b=24$ and 27, $c=45$ and 48, and $y=\frac{1}{7}x$. Put $x=7$, then $y=1$, and each of the women received 50c or 30c.

NOTE.—A special case of Mr. Gruber's solution is to let y=the price they received for the eggs per dozen and x=the price they received for the remaining eggs. Then $10x$=amount the first woman received, $2y+6x$=amount the second received, and $4y+2x$=amount the third received. Since they all received the same amount, we have $10x=2y+6x=4y+2x$. Therefore $y=2x$. Hence, if they sell them at 1, 2, 3, or 4c each, and 2, 4, 6, or 8c per dozen, they will receive the same sum.

Mr. Charles C. Cross and Mr. M. E. Graber solved the problem by assuming that they sell 7 eggs for a cent and the remaining eggs at 3 cents each. Thus each woman would get 10 cents.

Professor Zerr assumes that the first woman sells 1 egg for 1 cent and the remaining 9 at 6 cents each, receiving, therefore, 55 cents; the second sells 25 eggs for 1 cent each and 5 eggs at 6 cents each; and the third 49 eggs at 1 cent each and 1 egg for 6 cents. This way each would receive 55 cents. ED. F.

ALGEBRA.

161. Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, England.

If n quantities are made up of q sets of r each, find the number of permutations s at a time. It is supposed that the quantities in each set are alike, but different from those in the other sets.

Solution by G. B. M. ZERR, A.M., Ph.D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

If all different, the number of permutations$=n!$; but r things can be permuted in $r!$ ways, and q sets of r things in a set, can be permuted in $(r!)^q$ ways.

$\therefore s\times(r!)^q=n!$, or $s=\dfrac{n!}{(r!)^q}$.

162. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

If $x=\Sigma_0^\infty e^{-k[t+(2a\pi/h)]}\sin n\left(t+\dfrac{2a\pi}{h}\right)$, find value of x freed from Σ_0^∞.

Solution by the PROPOSER.

Let $2\pi/h=m$. $\therefore x=\Sigma_0^\infty e^{-k(t+am)}\sin n(t+am)$. a can have all positive integral values.

Let $C=\Sigma_0^\infty e^{-akm}\cos(anm)$, $S=\Sigma_0^\infty e^{-akm}\sin(anm)$.

Then $x=e^{-kt}(C\sin nt+S\cos nt)$. Now $C+S\sqrt{(-1)}=\Sigma_0^\infty e^{-am(k+n\sqrt{-1})}$

$$=\frac{1}{1-e^{-m(k+n\sqrt{-1})}}=\frac{1}{1-e^{-km}\cos(mn)-\sqrt{(-1)}^{-km}\sin(mn)}$$

$$=\frac{1-e^{-km}\cos(mn)+\sqrt{(-1)}e^{-km}\sin(mn)}{1-2e^{-km}\cos(mn)+e^{-2km}}.$$

$$\therefore C=\frac{1-e^{-km}\cos(mn)}{1-2e^{-km}\cos(mn)+e^{-2km}},\quad S=\frac{e^{-km}\sin(mn)}{1-2e^{-km}\cos(mn)+e^{-2km}}.$$

$$\therefore x=\frac{e^{-kt}\{[1-e^{-km}\cos(mn)]\sin nt+e^{-km}\sin(mn)\cos nt\}}{1-2e^{-km}\cos(mn)+e^{-2km}}$$

$$=\frac{e^{-kt}-e^{-k(m+t)}\sin n(t-m)}{1-2e^{-km}\cos(mn)+e^{-2km}}.$$

NOTE ON PROBLEM 145 (UNSOLVED) BY H. S. VANDIVER, STUDENT, UNIVERSITY OF PENNSYLVANIA, PHILADELPHIA, PA.

It is possible to show that

$$F(a,b,c,d)=2b^2c^2+2c^2a^2+2a^2b^2+2a^2d^2+2b^2d^2+2c^2d^2-a^4-b^4-c^4-d^4$$

cannot be expressed as the product of two rational factors. For, assuming that we have

$$F(a,b,c,d)=f(a,b,c,d)f'(a,b,c,d)$$

(by symmetry both f and f' must contain all the letters a, b, c, and d). Put $a=b$, $c=d$. Then